On Independence and SCC-Recursiveness in Assumption-Based Argumentation (Extended Abstract)*

Lydia Blümel[1] , **Anna Rapberger**[2] , **Matthias Thimm**[1] and **Francesca Toni**[2]
[1]Artificial Intelligence Group, University of Hagen
[2]Imperial College London
{lydia.bluemel, matthias.thimm}@fernuni-hagen.de, {a.rapberger,ft}@imperial.ac.uk

1 Introducing Independence to ABA

Determining conditional (in)dependence between variables is an important concern in AI. The ability to recognize and manage (in)dependence is crucial in symbolic reasoning (Darwiche 1997; Lang, Liberatore, and Marquis 2002). In the remainder of this extended abstract we give an overview on our work (Blümel et al. 2025), where we study conditional independence in assumption-based argumentation.

Assumption-based argumentation (ABA) (Bondarenko et al. 1997) is a well-known form of computational argumentation, whose building blocks are assumptions (defeasible elements) and inference rules. In situations where some assumptions cannot be true together, ABA is a suitable reasoning tool. ABA is widely applicable, e.g., to provide explanations (Cyras, Heinrich, and Toni 2021) and for causal discovery (Russo, Rapberger, and Toni 2024). In all these settings, a good understanding of independence between various components in ABA frameworks (ABAFs) is crucial. Based on a deductive system $(\mathcal{L}, \mathcal{R})$, where $\mathcal{L}$ is a set of sentences and $\mathcal{R}$ a set of inference rules, an ABAF is defined as a tuple $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, ^-)$, where $\mathcal{A}$ is a set of assumptions from which inferences are drawn using rules in $\mathcal{R}$, and $^- : \mathcal{A} \to \mathcal{L}$ is a contrary function used to derive attacks on assumptions. In order to conduct defeasible reasoning in ABA, we consider *arguments* that can be built by applying rules to assumptions. More precisely, we call a derivation of some $p \in \mathcal{L}$ from a set of assumptions $A \subseteq \mathcal{A}$ via rules in $\mathcal{R}$ an *(ABA) argument* if each assumption is used in the derivation, and denote this by $A \vdash p$. Furthermore, if p is the contrary $\overline{b}$ of some assumption $b \in \mathcal{A}$ we say the set A *attacks* any set $B \subseteq \mathcal{A}$ with $b \in B$. A set of assumptions A is conflict-free if it does not attack itself; admissible if it is conflict-free and defends itself, i.e., attacks all of its attackers. On top of this, semantics for ABAFs can be defined using labellings, which assign one of the three labels `in`, `out`, or `und` to each assumption. A *labelling-based semantics* σ assigns to each ABAF D a set of σ-labellings $\Lambda_\sigma(D)$.

*****Acknowledgements** The research reported here was partially supported by the Deutsche Forschungsgemeinschaft (grant 550735820), by ERC under the EU's Horizon 2020 research and innovation programme (grant agreement no. 101020934, ADIX), and by J.P. Morgan and the Royal Academy of Engineering under the Research Chairs and Senior Research Fellowships scheme (grant agreement no. RCSRF2021\11\45).

For our following example we recall the labelling-based version of preferred semantics (Schulz and Toni 2017). A labelling on some ABAF D is preferred, if sets of `in`-labeled assumptions only attack `out`-labeled assumptions(conflict-free), every set attacking an `in`-labeled assumption contains an `out`-labeled assumption (defense) and the set of `in`-labeled assumptions is maximal among all labellings satisfying these two conditions (maximality).

Example 1.1. *Let us consider an ABAF D with assumptions $\{a, b, c, d, w, k\}$, their contraries, and for each $v \in \{a, b, c\}$, the inference rules ($\overline{v} \leftarrow d$), ($\overline{d} \leftarrow v$), and further rules*

$$\overline{a} \leftarrow b, c \quad \overline{b} \leftarrow a, c \quad \overline{c} \leftarrow a, c \quad \overline{k} \leftarrow a \quad \overline{w} \leftarrow d$$

The ABAF D has four preferred labellings.

pr-lab	*a*	*b*	*c*	*d*	*w*	*k*
λ_1	*in*	*in*	*out*	*out*	*in*	*out*
λ_2	*in*	*out*	*in*	*out*	*in*	*out*
λ_3	*out*	*in*	*in*	*out*	*in*	*in*
λ_4	*out*	*out*	*out*	*in*	*out*	*in*

Having defined the setting, we are now ready to formulate the topic of our investigation: *under what conditions are two (sets of) assumptions independent from each other, relative to a (potentially empty) set of assumptions that is considered prior knowledge?*

Motivated by the work (Rienstra et al. 2020) on conditional independence in abstract argumentation, we identify (in)dependencies between sets of assumptions in ABA through the compatibility of partial labellings on them. For $A \subseteq \mathcal{A}$, we let $\lambda|_A : A \to \{\texttt{in}, \texttt{out}, \texttt{und}\}$ denote the restriction of some labelling λ to a partial labelling on A.

Definition 1.2. *Let $D = (\mathcal{L}, \mathcal{R}, \mathcal{A}, ^-)$ be an ABAF, σ a semantics, and $A, B, C \subseteq \mathcal{A}$ disjoint sets of assumptions.* A is σ-independent of B, given C in D, *written $A \perp_\sigma B \mid C$ iff, for all labellings $\lambda_1, \lambda_2 \in \Lambda_\sigma(D)$, it holds that if $\lambda_1|_C = \lambda_2|_C$ then there is some labelling $\lambda_3 \in \Lambda_\sigma(D)$ s.t. $\lambda_3|_A = \lambda_1|_A$,$\lambda_3|_B = \lambda_2|_B$, and $\lambda_3|_C = \lambda_1|_C = \lambda_2|_C$.*

Intuitively, a set of assumptions A is dependent on another set B under a semantics σ, if knowing the labelling on B excludes some possible σ-labellings on A. Indeed, this labelling-based independence notion allows us to derive reasonable (in)dependencies for our example.

Example 1.3. *The assumptions a and b are σ-independent wrt. $\emptyset$ for $\sigma = pr$ in the ABAF D. For this, we identify the labels which are individually assigned to a and b under σ, i.e., a and b can both be labeled* out *at the same time (λ_4). However, when conditioning on $\{c\}$, the assumptions a and b are dependent under pr semantics. If $\lambda(c) = \mathtt{in}$, then a and b can be individually* in*, but not together.*

2 Overview of Results

In (Blümel et al. 2025), we focus on a well-studied fragment of ABA whereby assumptions cannot be inferred (*flat* ABA) and where assumptions, their contraries and any sentences in inference rules are atomic. We provide complexity results for deciding independence in both ABA and abstract argumentation frameworks (Rienstra et al. 2020) and propose an SCC-recursive schema for ABA semantics alongside sound polynomial time checks for independence between sets of assumptions. Below, we outline our main contributions.

We introduce conditional independence to ABA as a means to analyze the relations between the acceptance of disjoint sets of assumptions. We say two sets of assumptions A and B are independent, given a third set C, if, once we know the labellings of the assumptions in C (in, out, und), then the possible labellings for A are no longer influenced by those for B. This notion, captured by Definition 1.2, allows us to identify when reasoning about one part (e.g., A) can be done independently of another (e.g., B). We show our definition behaves as expected and satisfies the semi-graphoid axioms. The complexity of deciding independence, however, turns out to be rather high.

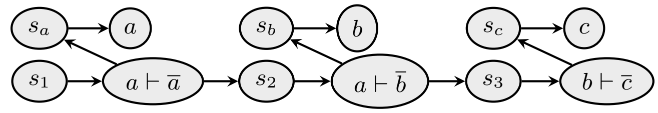
Proposition 2.1. *Let $\mathcal{C}$ denote the class of AFs/flat ABAFs. Deciding σ-independence in $\mathcal{C}$ is Π_2^P-complete for $\sigma \in \{co, st\}$ and Π_3^P-complete for $\sigma = pr$.*

We show a correspondence to conditional independence for AFs (Rienstra et al. 2020) and settle the complexity of the corresponding decision problems for those.

To alleviate the high computational complexity we exploit the structure of ABA and explore *SCC-recursiveness* for ABAFs. SCC-recursiveness is well-studied in the realm of abstract argumentation (Baroni, Giacomin, and Guida 2005; Dvorák et al. 2024); semantics that satisfy this property can be processed locally, along the *strongly connected components (SCCs)* of a graph. We construct an SCC-recursive scheme for ABAFs around the defeasible rules, which are the core of reasoning in ABA, and provide SCC-recursive characterizations of the standard semantics.

For our SCC-based independency checks we first examine whether we can exploit the close connection between AFs and ABAFs to use the $\mathbb{P}$-time check for independence within AFs proposed by (Rienstra et al. 2020) for ABAFs. The method utilizes the SCC-structure of the induced AF to facilitate a check via d-separation which requires a DAG (directed acyclic graph). For that we use the d-graph of the AF, which has a tree-structure and contains a node for each SCC of the AF, having the contained arguments as children, from which in turn other SCC-nodes are reached. The approach is illustrated in the following example.

Example 2.2. *Consider an ABAF D with assumptions a, b, c and rules $(\overline{a} \leftarrow a), (\overline{b} \leftarrow a), (\overline{c} \leftarrow b)$. All assumptions of the d-graph G_{F_D} of AF F_D for ABAF D are in terminal SCCs.*



The AF-SCCs that contain assumption arguments are often either terminal or initial SCCs. For a sensitive independence check we require an SCC-structure which is informative enough. We propose an alternative check using directly the SCC-structure of the ABAF. The SCC-decomposition of an ABAF is conducted via the dependency graph P_D (Rapberger, Ulbricht, and Wallner 2022). We can now compute the d-graph G_D with respect to said dependency graph and check for independencies using the d-separation criterion. This allows insights on the (in)dependencies between assumptions the AF-based approach cannot procure.

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